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Circuit Design Criteria For Stability In A Class Of Lateral Inhibition Neural Networks

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Abstract

In the analog VLSI implementation of neural systems, it is sometimes convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. A serious problem of unwanted spontaneous oscillation often arises with these circuits and renders them unusable in practice. This paper reports a design approach that guarantees such a system will be stable, even though the values of designed elements in the resistive grid may be imprecise and the location and values of parasitic elements may be unknown. The method is based on a mathematical analysis using Tellegen's theorem and the Popov criterion. The criteria are *local* in the sense that no overall analysis of the interconnected system is required for their use, *empirical* in the sense that they involve only measurable frequency response data on the individual cells, and *robust* in the sense that they are not affected by unmodelled parasitic resistances and capacitances in the interconnect network.

I. Introduction

The term "lateral inhibition" first arose in neurophysiology to describe a common form of neural circuitry in which the output of each neuron in some population is used to inhibit the response of each of its neighbors. Perhaps the best understood example is the horizontal cell layer in the vertebrate retina, in which lateral inhibition simultaneously enhances intensity edges and acts as an automatic gain control to extend the dynamic range of the retina as a whole [1]. The principle has been used in the design of artificial neural system algorithms by Kohonen [2] and others and in the electronic design of neural chips by Carver Mead et. al. [3,4].

In the VLSI implementation of neural systems, it is convenient to build lateral inhibition networks by using a locally connected on-chip resistive grid. Linear resistors fabricated in, e.g., polysilicon, could yield a very compact realization, and nonlinear resistive grids, made from MOS transistors, have been found useful for image segmentation [4,5]. Networks of this type can be divided into two classes: feedback systems and feedforward-only systems. In the feedforward case one set of amplifiers imposes signal voltages or currents on the grid and another set reads out the resulting response for subsequent processing, while the same amplifiers both "write to" the grid and "read from" it in a feedback arrangement. Feedforward networks of this type are inherently stable, but feedback networks need not be.

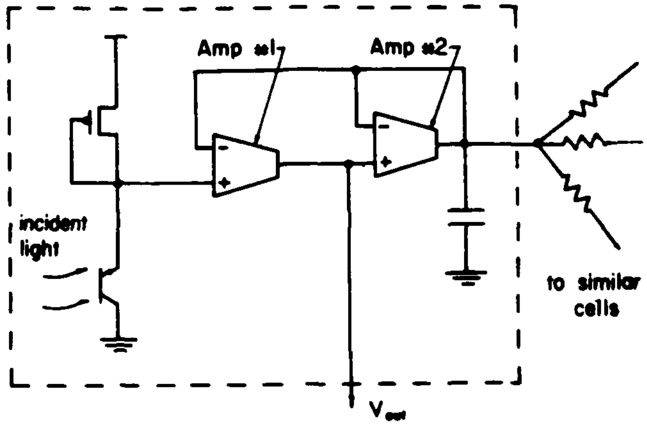

Figure 1: This photoreceptor and signal processor circuit, using two MOS amplifiers, realizes lateral inhibition by communicating with similar cells through a resistive grid.

A practical example is one of Carver Mead's retina chips [3] that achieves edge enhancement by means of lateral inhibition through a resistive grid. Figure 1 shows a single cell in a continuous-time version of this chip, and Fig. 2 illustrates the network of interconnected cells. Note that the voltage on the capacitor in any given cell is affected both by the local light intensity incident on that cell and by the capacitor voltages on neighboring cells of identical design. Each cell drives its neighbors, which drive both their distant neighbors and the original cell in turn. Thus the necessary ingredients for instability — active elements and signal feedback — are both

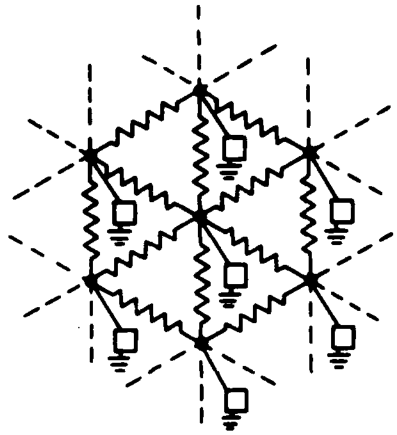

Figure 2: Interconnection of cells through a hexagonal resistive grid. Cells are drawn as 2-terminal elements with the power supply and signal output lines suppressed. The grid resistors will be nonlinear by design in many such circuits.

present in this system. Experiment has shown that the individual cells in this system are open-circuit stable and remain stable when the output of amp # 2 is connected to a voltage source through a resistor, but the interconnected system oscillates so badly that the original design is essentially unusable in practice with the lateral inhibition paths enabled [6]. Such oscillations can readily occur in most resistive grid circuits with active elements and feedback, even when each individual cell is quite stable. Analysis of the conditions of instability by conventional methods appears hopeless, since the number of simultaneously active feedback loops is enormous.

This paper reports a practical design approach that rigorously guarantees such a system will be stable. The work begins with the naivé observation that the system would be stable if we could design each individual cell so that, although internally active, it acts like a passive system as seen from the resistive grid. The design goal in that case would be that each cell's output impedance should be a *positive-real* [7,8, and 9, p. 174] function. This is sometimes possible in practice; we will show that the original network in Fig. 1 satisfies this condition in the absence of certain parasitic elements. Furthermore, it is a condition one can verify experimentally by frequency-response measurements.

It is obvious that a collection of cells that appear passive at their terminals will form a stable system when interconnected through a passive medium such as a resistive grid, and that the stability of such a system is *robust* to perturbations by passive parasitic elements in the network. The contribution of this paper is to go beyond that observation to provide i) a demonstration that the passivity or positive-real condition is much stronger than we actually need and that weaker conditions, more easily

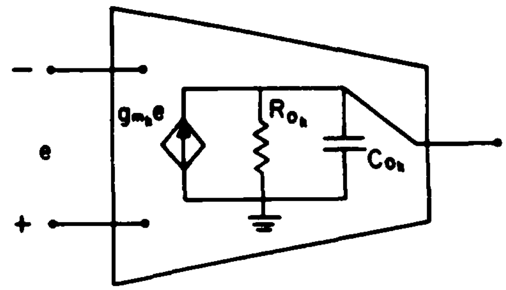

Figure 3: Elementary model for an MOS amplifier. These amplifiers have a relatively high output resistance, which is determined by a bias setting (not shown).

achieved in practice, suffice to guarantee robust stability of the linear network model, and ii) an extension of the analysis to the *nonlinear* domain that furthermore rules out sustained *large-signal* oscillations under certain conditions.

Note that the work reported here does not apply directly to networks created by interconnecting neuron-like elements, as conventionally described in the literature on artificial neural systems, through a resistive grid. The "neurons" in, e.g., a Hopfield network [10] are *unilateral 2-port elements* in which the input and output are both voltage signals. The input voltage uniquely and instantaneously determines the output voltage of such a neuron model, but the output can only affect the input via the resistive grid. In contrast, the cells in our system are *1-port electrical elements* (temporarily ignoring the optical input channel) in which the port voltage and port current are the two relevant signals, and each signal affects the other through the cell's internal dynamics (modelled as a Thevénin equivalent impedance) as well as through the grid's response.

II. The Linear Theory

This work was motivated by the following linear analysis of a model for the circuit in Fig. 1. For an initial approximation to the output admittance of the cell we use the elementary model shown in Fig. 3 for the amplifiers and simplify the circuit topology within a single cell (without loss of relevant information) as shown in Fig. 4.

Straightforward calculations show that the output admittance is

$$Y(s) = [g_{m_2} + R_{o_2}^{-1} + sC_{o_2}] + \frac{g_{m_1} g_{m_2} R_{o_1}}{(1 + sR_{o_1}C_{o_1})}, \quad (1)$$

which is positive-real.

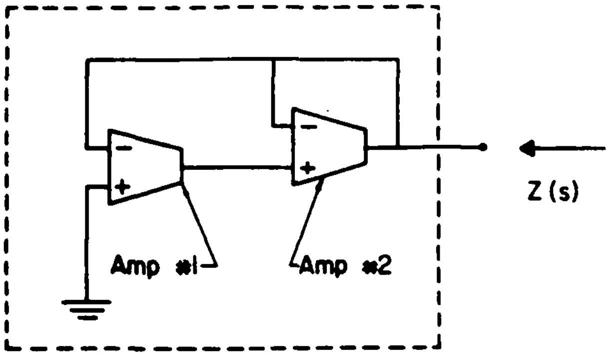

Figure 4: Simplified network topology for the circuit in Fig. 1. The capacitor that appears explicitly in Fig. 1 has been absorbed into C_{o_2}.

Of course this model is oversimplified, since the circuit *does* oscillate. Transistor parasitics and layout parasitics cause the output admittance of the individual active cells to deviate from the form given in eq. (1), and any very accurate model will necessarily be quite high order. The following theorem shows how far one can relax the positive-real condition and still guarantee that the entire network is robustly stable.

Terminology

The terms *open right-half plane* and *closed right-half plane* refer to the set of all complex numbers $s = \sigma + j\omega$ with $\sigma > 0$ and $\sigma \geq 0$, respectively, and the term *closed second quadrant* refers to the set of complex numbers with $\sigma \leq 0$ and $\omega \geq 0$. A *natural frequency* of a linear network is a complex frequency s_o such that, when all independent sources are set to zero and all branch impedances and admittances are evaluated at s_o, there exists a nonzero solution for the complex branch voltages $\{V_k\}$ and currents $\{I_k\}$ [11]. A lumped linear network is said to be *stable* if a) it has no natural frequencies in the closed right-half plane except perhaps at the origin, and b) any natural frequency at the origin results only in network solutions that are constant as functions of time. (The latter condition rules out unstable transient solutions that grow polynomially in time resulting from a repeated natural frequency at the origin.)

Theorem 1

Consider the class of linear networks of arbitrary topology, consisting of any number of positive 2-terminal resistors and capacitors and of N lumped linear impedances $Z_n(s), n = 1, 2, ..., N$, that are open- and short-circuit stable in isolation, i.e., that have no poles or zeroes in the closed right-half plane. Every such network is *stable* if at each frequency $\omega \geq 0$ there exists a phase angle $\theta(\omega)$ such that $0 \geq \theta(\omega) \geq -90°$ and $|\angle Z_n(j\omega) - \theta(j\omega)| < 90°, n = 1, 2, ..., N$.

An equivalent statement of this last condition is that the Nyquist plot of each cell's output impedance for $\omega \geq 0$ never intersects the closed 2nd quadrant, and that no two cells' output impedance phase angles can ever differ by as much as 180°. If all the active cells are designed identically and fabricated on the same chip, their phase angles should track fairly closely in practice, and thus this second condition is a natural one.

The theorem is intuitively reasonable. The assumptions guarantee that the cells cannot resonate with one another at any purely sinusoidal frequency $s = j\omega$ since their phase angles can never differ by as much as 180°, and they can never resonate with the resistors and capacitors since there is no $\omega \geq 0$ at which both $Re\{Z_n(j\omega)\} \leq 0$ and $Im\{Z_n(j\omega)\} \geq 0$ for some $n, 1 \leq n \leq N$. The proof formalizes this argument using conservation of complex power, extends it to rule out natural frequencies in the right-half plane as well, and shows why instabilities resulting from a repeated natural frequency at the origin cannot occur.

Proof of Theorem 1

Let s_o denote a natural frequency of the network and $\{V_k\}, \{I_k\}$ denote any complex network solution at s_o. By Tellegen's theorem [12], or conservation of complex power, we have

$$\sum_k V_k I_k^* = 0, \tag{2}$$

i.e., for $s_o \neq$ any pole of $Z_n, n = 1, ..., N$ and $s_o \neq 0$,

$$\sum_{\text{resistors}} |I_k|^2 R_k + \sum_{\text{capacitors}} |I_k|^2 (s_o C_k)^{-1} + \sum_{\text{cells}} |I_n|^2 Z_n(s_o) = 0 \tag{3}$$

and for $s_o \neq$ any zero of $Z_n, n = 1, ..., N$,

$$\sum_{\text{resistors}} |V_k|^2 R_k^{-1} + \sum_{\text{capacitors}} |V_k|^2 s_o^* C_k + \sum_{\text{cells}} |V_k|^2 Y_n^*(s_o) = 0 \tag{4}$$

where the superscript $*$ denotes the complex conjugate operation. The proof is completed in the following three parts, which together rule out the existence of any natural frequencies in the closed right-half plane (except possibly for a single one at the origin).

Part i)

This part shows that there are no natural frequencies at $s_o = j\omega \neq 0$. For each $\omega > 0$ all the cell impedance values

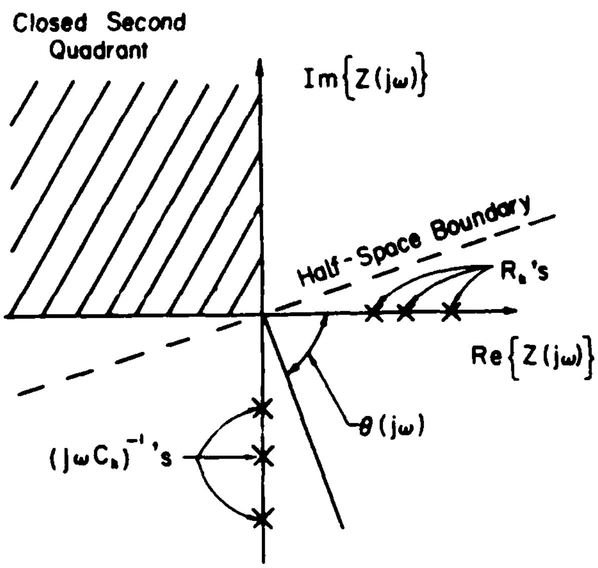

Figure 5: Illustration for the proof of Theorem 1.

lie strictly below and to the right of a half-space boundary passing through the origin of the complex plane at an angle $\theta(\omega) + 90°$ with the real positive axis, as shown in Fig. 5. The capacitor impedances $\{(j\omega C_k)^{-1}\}$ and the resistor impedances $\{R_k\}$ also lie below and to the right of this line. Thus no positive linear combination of these impedances can vanish as required by (3). A similar argument holds for $\omega < 0$.

Part ii)

This part shows that there cannot exist a repeated natural frequency at the origin that leads to a time-dependent solution. The assumptions that the cell impedances have no $j\omega$-axis zeroes and that their Nyquist plots for $\omega \geq 0$ never intersect the closed 2nd quadrant imply that $Y_n^*(0) > 0$, $n = 1, ..., N$. Thus (4) requires that all the voltages across resistor branches and cell output branches must vanish in any complex network solution at $s_o = 0$. Thus only capacitor voltages can be nonzero and the network solution will be unaltered if all non-capacitor branches are replaced by short circuits. But every solution to a network comprised only of positive, linear 2-terminal capacitors is constant in time (and hence stable).

Part iii)

This part uses a homotopy argument to show that there are no natural frequencies in the open right-half plane. Assume the contrary, i.e., that there exists such a network with a natural frequency s_1 with $Re\{s_1\} > 0$. Alter each element in the network (except resistors) as follows. For each cell having a $Z_n(s)$ of relative degree less than zero, add a series resistance R; for all other cells and for capacitors, add a parallel conductance G to each. Call each resulting pair a "composite element", and choose $R = G = \lambda \geq 0$. For λ sufficiently large all natural frequencies must lie in the open left-half plane since every branch element is strictly passive for λ sufficiently large. Since the natural frequencies are continuous functions of λ [13] and $Re\{s_1\} > 0$ for $\lambda = 0$, there exists some $\lambda > 0$ for which some natural frequency s_1' lies on the imaginary axis. But this is ruled out by the proof in part i) unless $s_1' = 0$, and the argument in part ii) rules out $s_1' = 0$, since any network solution at $s_1' = 0$ consists of zero branch voltages except for capacitor branches, and for $\lambda > 0$ each capacitor has a positive conductance G in parallel with it. Since the voltage across every G is zero in such a network solution, all branch voltages (and thus all branch currents) in that solution must be zero, which is a contradiction because a natural frequency at s_1' implies the existence of a nonzero solution. ■

III. Stability Result for Networks with Nonlinear Resistors and Capacitors

The previous results for linear networks can afford some limited insight into the behavior of nonlinear networks. *If a linearized model is stable, then the equilibrium point of the original nonlinear network is locally stable.* But the result in this section, in contrast, applies to the full nonlinear circuit model and allows one to conclude that in certain circumstances the network cannot oscillate *even if* the initial state is *arbitrarily far from* the equilibrium point.

Terminology

We say that a function $y = f(x)$ *lies in the sector* [a,b] if $ax^2 \leq xf(x) \leq bx^2$. And we say that an impedance $Z(s)$ satisfies the *Popov criterion* if $(1 + \tau s)Z(s)$ is *positive real* [7,8,and 9, p. 174] for some $\tau > 0$. (Note that this formulation of the Popov criterion differs slightly from that given in standard references [8 and 9, p. 186].)

Theorem 2

Consider a network consisting of possibly nonlinear resistors and capacitors and cells with linear output impedances $Z_n(s), n = 1, 2, ..., N$. Suppose

i) the resistor curves are continuous functions $i_k = g_k(v_k)$ where g_k lies in the sector $[0, G_{max}], G_{max} > 0$, for all resistors,

ii) the capacitors are characterized by continuous functions $i_k = C_k(v_k)\dot{v}_k$ where $0 \leq C_k(v_k) \leq C_{max}$ for all k and v_k, and

iii) the impedances $Z_n(s)$ all satisfy the Popov criterion for some common value of $\tau > 0$. Then the network is stable in the sense that, for any initial condition,

$$\int_0^\infty \Big[\sum_{\substack{\text{all resistors} \\ \text{and capacitors}}} i_k^2(t) \Big] dt < \infty . \quad (5)$$

Outline of Proof

By Tellegen's theorem, for any set of initial conditions and any time $T > 0$,

$$\int_0^T \sum_{\text{resistors}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \; +$$

$$\int_0^T \sum_{\text{capacitors}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt \; +$$

$$\int_0^T \sum_{\text{cell impedances}} (v_k(t) + \tau \dot{v}_k(t)) i_k(t) dt = 0. \quad (6)$$

For resistors, multiplying the sector inequality $vg(v) \leq G_{\max} v^2$ by $\frac{i}{v} \geq 0$ yields $i^2 = ig(v) \leq G_{\max} iv$, and hence

$$G_{\max}^{-1} \int_0^T i_k^2(t) dt \leq \int_0^T i_k(t) v_k(t) dt =$$

$$\int_0^T i_k(t)[v_k(t) + \tau \dot{v}_k(t)] dt - \tau[\phi_k(v_k(T)) - \phi_k(v_k(0)] \quad (7)$$

where

$$\phi_k(v) = \int_0^v g_k(v') dv' \geq 0 \quad (8)$$

is the *resistor co-content.* Using the inequality (8) in (7) yields for each resistor

$$G_{\max}^{-1} \int_0^T i_k^2(t) dt - \tau \phi_k(v_k(0)) \leq \int_0^T i_k(t)[v_k(t) + \tau \dot{v}_k(t)] dt. \quad (9)$$

For capacitors, integrating the inequality $i_k^2 = C_k^2(v_k)\dot{v}_k^2 \leq C_{\max} C_k(v_k) \dot{v}_k^2$ yields

$$\frac{\tau}{C_{\max}} \int_0^T i_k^2(t) dt \leq \tau \int_0^T C_k(v_k) \dot{v}_k^2(t) dt =$$

$$\int_0^T i_k(t)[v_k(t) + \tau \dot{v}_k(t)] dt - [E_k(q_k(T) - E_k(q_k(0)], \quad (10)$$

where

$$E_k(q) = \int_0^q v_k(q') dq' \geq 0 \quad (11)$$

is the capacitor energy. Using the inequality (11) in (10) yields for each capacitor

$$\frac{\tau}{C_{\max}} \int_0^T i_k^2(t) dt - E_k(q_k(0) \leq \int_0^T i_k(t)[v_k(t) + \tau \dot{v}_k(t)] dt. \quad (12)$$

And for the cells, the assumption that $(1 + \tau s) Z_n(s)$ is positive real implies that

$$\int_0^T i_n(t)[v_n(t) + \tau \dot{v}_n(t)] dt \geq -E_n(0), \quad (13)$$

where $E_n(0)$ is the initial "energy" in the mathematically constructed impedance $(1 + \tau s) Z_n(s)$ at $t = 0$, a function of the initial conditions only. Substituting (9), (12) and (13) into (6) yields

$$G_{\max}^{-1} \int_0^T \sum_{\text{resistors}} i_k^2(t) dt + \frac{\tau}{C_{\max}} \int_0^T \sum_{\text{capacitors}} i_k^2(t) dt \leq$$

$$\tau \sum_{\text{resistors}} \phi_k(v_k(0)) + \sum_{\text{capacitors}} E_k(q_k(0)) + \sum_{\text{cells}} E_n(0), \quad (14)$$

where the right hand side is a function only of the initial conditions. Thus (5) holds. ∎

Note that Thm. 2, as it is stated, applies only to networks in which the voltage source waveform of each cell's Thevénin equivalent circuit is identically zero. In practice, these voltages are generally nonzero and change with time. Yet a necessary condition for design is that the circuit be stable for *constant* Thevénin voltages (which would result from a constant light input). If this condition is met, then the effect of time variation can be thought of as an issue separate from stability and related to the convergence rate of the network towards a "time-dependent equilibrium point." Thus, it is appropriate to extend Thm. 2 to include the case of cells that have arbitrary but constant Thevénin voltages. This can be done simply by requiring the resistor curves to satisfy the sector condition i) of the theorem about all possible equilibrium points. Even if there is no known restriction on the set of equilibrium points, the sector condition will be satisfied at every equilibrium point if all the g_k's are nondecreasing differentiable functions with bounded slope.

IV. Concluding Remarks

The design criteria presented here are simple and practical, though at present their validity is restricted to linear models of the cells. There are several areas of further work to be pursued, one of which is an analysis of the cell that includes amplifier clipping effects. Others include the synthesis of a compensator for the cell, an extension of the nonlinear result to include impedance multipliers other than the Popov operator, a bound on the network settling time when the optical input is constant, and a

bound on the L_2 norm of the resistor and capacitor currents in terms of the L_2 norm of the Thevénin equivalent cell voltage waveforms when the optical input is time-varying.

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